Questions on Random Matrix Theory and Cohen-Lenstra Heuristics

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1 Questions

1.1 Question 1

What is the asymptotic probability that a Bernoulli random matrix over a finite field has a cyclic vector?

1.2 Question 2

What is the asymptotic probability that the determinant of a Bernoulli random matrix over $\mathbb{Z}$ is square-free?

1.3 Question 3

Define the following function $F_B(G)$ on finite abelian p-groups. If $G \cong G' \times B$, let $F_B(G) = \frac{\#Aut(G)}{\#Aut(G')}$ and $F_B(G) = 0$ if G does not have B as a factor. We have the identity

$$F_{\mathbb{Z}/p\mathbb{Z}}(G) = \frac{1}{p}\Big(\#Hom(G, \mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z}) - \#Hom(G, \mathbb{Z}/p^2\mathbb{Z})\Big)$$

I believe that $F_B(G)$ is always a finite linear combination of functions of the form $Hom(G, A)$, where A varies over groups of cardinality at most $(\#B)^2$. Is there a group-theoretic proof of this? What are the coefficients of $Hom(G, A)$ that appear in the expansion of $F_B(G)$?

Remark. The function $F_B(G)$ is related to the number of split surjections from G to B.

1.4 Question 4

What is the asymptotic distribution of the cokernel of $XY - YX$? Is it Cohen-Lenstra when the matrices X, Y are Haar random?

1.5 Question 5

If G is a Haar random element of a classical group (such as the symplectic or orthogonal group), and f is a monic polynomial, what is distribution of $coker(f(G))$?

1.6 Question 6

What is the optimal convergence rate of the cokernel of a Bernoulli random matrix over $\mathbb{Z}_p$ to the Cohen-Lenstra distribution? What is the best metric to measure this rate, over $\mathbb{Z}_p$? Perhaps some sort of Wasserstein distance between measures?

Remark. There is a natural distance function on the set of finite abelian p-groups: $d(G, H) = p^{-k}$ if p^k is the largest power of p modulo which G and H are isomorphic. This gives a family of P-Wasserstein distances between probability measures.

1.7 Question 7

(Inspired by Roger's work.) Consider the following process over a finite local ring R with residue field k. Start with the 0 module. At each case, take a random k-extension of the module at the previous step.

Now say that two modules are in the same equivalence class when $M \times R^m \cong N \times R^n$ for some m and n. What kind of distribution would one get on equivalence classes of modules?

Does this distribution arise in cokernels of random products over R. Is it universal?

1.8 Question 8

A random walk similar to the "Cohen-Lenstra" random walk can naturally be defined for groups with a perfect symmetric/anti-symmetric pairing, groups equipped with a surjection to a given group B, ... Is it possible that there is a general definition of a random walk on categories, that encompasses if not all, then many of these cases?

1.9 Question 9

How to parametrize $GL(\mathbb{Z}_p)$ orbits of tensors over $\mathbb{Z}_p$? Are Bhargava's, Wood's parametrizations useful for this?

Try to generalize some aspects of random matrix theory to tensors. Maybe start with $2 \times n \times n+1$ tensors. In this case, over a finite field, all tensors with non-zero hyper-determinant lie in the same $GL_2 \times GL_n \times GL_{n+1}$ orbit (which is also the case for matrices). The probability of the hyper-determinant being non-zero has been computed by Colin Aitken to be

$$\prod_{i=1}^{n}\left(1-\frac{1}{q^i}\right)\prod_{j=2}^{n+1}\left(1-\frac{1}{q^j}\right)$$

The result is reminiscent of the corresponding result in the matrix case, Are there any other similarities to the random matrix theory in this case?

Remark. $GL(Z_p)$ orbits of "corners" of Haar random tensors also have a Markovian property.

What is the asymptotic probability that the hyperdeterminant of a tensor is divisible by m?